# Magnetic field asymmetry of non-linear transport in Aharonov Bohm rings

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Fundamental Casimir-Onsager symmetry rules for linear response do not apply to non linear transport. This motivates the investigation of nonlinear dc conductance of mesoscopic GaAs/GaAlAs rings in a 2 wire configuration. The second order current response to a potential bias is of particular interest. It is related to the sensitivity of conductance fluctuations to this bias and contains information on electron interactions not included in the linear response. In contrast with the linear response which is a symmetric function of magnetic field we find that this second order response exhibits a field dependence which contains an antisymmetric part. We analyse the flux periodic and aperiodic components of this asymmetry and find that they only depend on the conductance of the rings which is varied by more than an order of magnitude. These results are in good agreement with recent theoretical predictions relating this asymmetric response to the electron interactions.

Magneto-transport properties of mesoscopic systems are sensitive to interferences between electronic waves and exhibit characteristic signatures of phase coherence. [1] Among them are universal conductance fluctuations (UCF) leading to reproducible sample specific magnetoresistance patterns, which in a ring geometry are modulated by the flux periodic Aharonov-Bohm (AB) oscillations. Beside these effects on the linear conductance, it has been shown that mesoscopic systems exhibit rectifying properties related to the absence of spatial inversion symmetry of the disorder or confining potential. This gives rise to a quadratic term in the $I, V$ relation: $I = G_1 V + G_2 V^2$. This non linearity was predicted theoretically [2] and observed experimentally more than 15 years ago [3]. It was understood as a direct consequence of the sensitivity of conductance fluctuations to the Fermi energy with a characteristic scale given by the Thouless energy, $E_c = h/\tau_D$, where $\tau_D = L^2/D$ is the diffusion time across the sample of size $L$. More recently it has been pointed out that non linear transport coefficients do not obey Casimir Onsager symmetry rules [4] and may include, in a 2 wires measurement, a component antisymmetric in magnetic field with a linear dependence at low field. On the experimental side the existence of a term linear both in field and current was found in macroscopic helical structures and attributed to magnetic self inductance effects [5]. It was subsequently observed in carbon nanotubes [6] and suggested to be related to their helical structure. More generally, this field asymmetry of non linear transport has been theoretically shown to be related to electron-electron interactions both in chaotic [7] and diffusive [8] systems. At the single impurity level, this effect can be simply viewed as the modification of the electron density $dn(\vec{r})$ around a scatterer in the presence of a current through the sample [9, 10]. Due to Coulomb interactions this results, in a modification of the potential around the impurity by a component linear in current. In a phase coherent sample, this bias induced change of disorder potential $dU_{dis}$ modifies the conductance fluctuations giving rise to the nonlinear conductance $G_2$ defined as: $G(V) = G_1 + G_2 V + ...$ Just like the chemical potential measured in a multiprobe transport measurement [1], $dU_{dis}$ exhibits field dependent fluctuations which have both symmetric and antisymmetric parts, including a component linear in B at low field. As a result $G_2$ has a symmetric component in $B$ and an antisymmetric one, respectively equal to $G_2^{S,AS} = (G_2(B) \pm G_2(-B))/2$ which both vary on the flux scale $\Phi_c$ characteristic of conductance fluctuations. The antisymmetric component only exists in the presence of electron-electron interactions. The typical amplitudes of these components have been calculated in a 2D system [7, 8, 11] and yield for weak interactions:

$$\delta G_2^S \sim \delta G_1 (e/E_c),\ \delta G_2^{AS} \sim \gamma_{int} \frac{\delta G_1}{g} f(|\Phi/\Phi_c|) \delta G_2^S \quad (1)$$

where the function $f(x)$ is equal to $x$ for $0 < x \ll 1$ and to 1 for $x \gg 1$. $g =< G_1 >$ and $\delta G_1 \simeq 1$ are the average conductance and the typical amplitude of $G_1$ fluctuations in units of $e^2/h$. In the weak interaction regime investigated in [8] $\gamma_{int} = 2\nu dU_{dis}(\vec{r})/dn(\vec{r}) \ll 1$, where $\nu$ is the density of states per unit surface. In the self-consistent treatment of Coulomb interaction [11, 12], the interaction constant is determined by the ratio of typical charging energy of the sample $\sim e^2/C$, and mean level spacing $\Delta$ as $\gamma_{int} = 1/(1 + C\Delta/2e^2)$ (the limit $\gamma_{int} \ll 1$ of Eq. 1 corresponds to $dU/dn = e^2/2C$)[13]. The $1/g$ factor in Eq.1 indicates that the field asymmetry should be easily detectable in low conductance samples where $g$ does not exceed 10, but is not observable in metallic mesoscopic

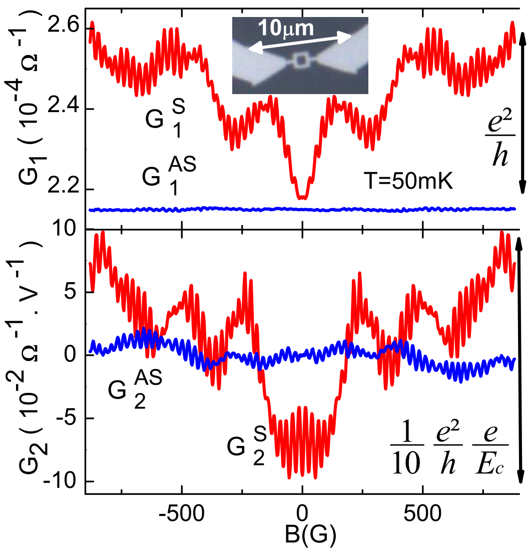


FIG. 1: Field dependence of $G_1^{S,AS}$ and $G_2^{S,AS}$,($G_1^{AS}$ shifted). These data were obtained on ring$_1$ in its initial state for a current of 10nA. Inset: micrograph of ring$_1$.

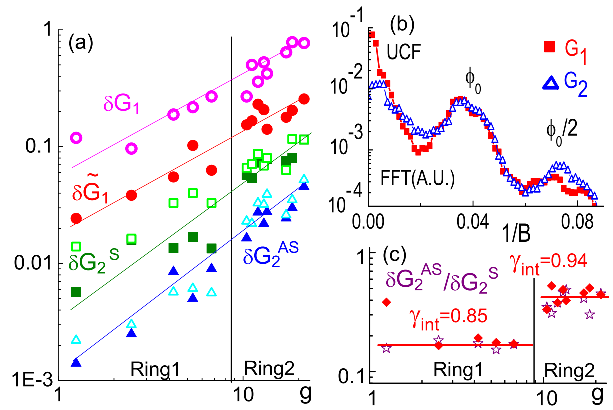


FIG. 2: (a)Amplitudes of the UCF and AB components of $G_1$ in units of $e^2/h$, $G_2^S$ and $G_2^{AS}$ in $(\Omega V)^{-1}$ as a function of the conductance of the rings. The open and closed symbols correspond to UCF and AB integrated peaks (straight lines are guides to the eyes). (b)Fourier transforms of $G_2$ and $G_1$ renormalised so that the amplitude of the AB peaks are identical. (c)Conductance dependence of the ratio $\delta G_2^{AS}/\delta G_2^S$ for UCF (open stars) in comparison with Eq.2 (diamonds), where $\gamma_{int} = 0.85$ for ring$_1$ and 0.94 for ring$_2$.

systems [14]. Carbon nanotubes [15] are in principle good candidates, but it is delicate to distinguish the effects due to the tube helicity from mesoscopic ones. Semiconducting samples are well suited for such investigations since they combine rather low conductance and large sensitivity to small fluxes [16, 17, 18]. We have measured the non linear conductance of 2 terminal GaAs/GaAlAs rings with only a few conducting channels. We find that $G_2$, like $G_1$, exhibits both AB oscillations and UCF conductance fluctuations. We show evidence of the existence of an asymmetry in magnetic field in $G_2$ from which we deduce the amplitude of the electron interactions.

The square rings investigated in this experiment were obtained by shallow etching through an aluminum mask, of a 2D electron gas (2DEG) of density $n_e = 3.8 \times 10^{15}\,\mathrm{m}^{-2}$ at the interface of a GaAs/GaAlAs heterojunction with Si donors. Contrary to previous experiments [15, 16, 17, 18], there is no electrostatic gate on the samples [19]. Due to depletion effects the real width of the rings is smaller than the etched one and is determined from magneto-transport data. We present data on 2 rings, of circumference $L = 4.8\mu m$ and respective width $W = 0.3,\ 0.45 \pm 0.05\mu m$. The elastic mean free path $l_e$ extracted from the conductance at 4K varies between 1 and $2\mu m$, which is less than the value of the initial 2DEG and comparable with the side of the square ring. In situ modifications of the samples were obtained by short illuminations with an electro-luminescent diode resulting in an increase of width and conductance of the rings. It was also possible to change the disorder configuration by applying current pulses in the 10 to 50 $\mu A$ range which decrease the average conductance of the samples. We could therefore, with only 2 samples, investigate a conductance ranging from $g = 1$ to 20. The measurements were conducted via filtered lines in a dilution refrigerator between 25mK and 1.2K. The samples were biased with ac current of frequency $\omega/2\pi \simeq 30Hz$ in the few nA range and voltage was measured with a low noise amplifier followed by lock-in amplifiers detecting the first and second harmonics response $V_1 \cos\omega t$ and $V_2 \cos 2\omega t$. The amplitude of the ac modulation was chosen to maximize the second harmonics signal in the regime where it is still quadratic with the current modulation amplitude. In this regime the second order conductance $G_2$ is simply related to $V_2$ and $V_1$ by $G_2 = -2V_2 I/V_1^3$. As shown in Fig.1 both $G_1(B)$ and $G_2(B)$ exhibit $h/e$ periodic AB oscillations modulated by UCF fluctuations.

More remarkable, whereas $G_1(B)$ is a symmetric function of magnetic field as expected in a 2 wires configuration, $G_2(B)$ exhibits a component antisymmetric in field $G_2^{AS}$. To compare the field asymmetry of $G_2$ on the various samples (Fig.2), we extract the amplitude of UCF and AB oscillations from the Fourier transform of $G_2^{S,AS}(B)$ and $G_1(B)$. The integrated UCF and AB peaks are noted $\delta G_2^{S,AS}$ and $\delta\tilde{G}_2^{S,AS}$ and similarly for $G_1$. These averages performed on a flux range much larger than $\Phi_c$ do not depend on this range. We find that all these quantities are aligned on logarithmic plots as a function of $g$. We first note the relatively large amplitudes of the AB oscillations $\delta\tilde{G}_2^{S,AS}$ which are of the order of the UCF components $\delta G_2^{S,AS}$ in contrast with the related quantities in $G_1$. The decrease of $\delta G_1$ and $\delta\tilde{G}_1$ to values much below 1 at low $g$ seems to be at odd with the universal character of conductance fluctuations in $G_1$ established deep in the diffusive regime $g \gg 1$. However this universal regime is not expected for $g \simeq 1$. The even

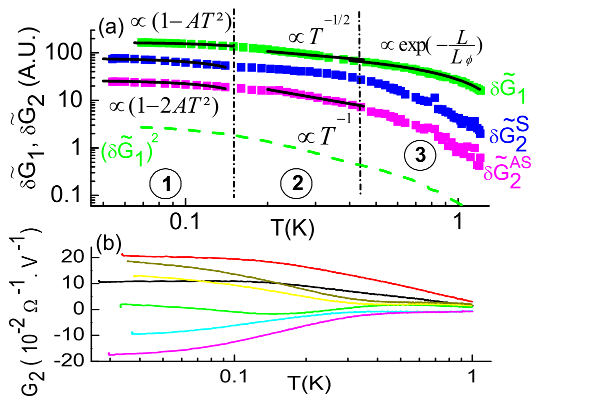


FIG. 3: (a)Temperature dependence of the amplitude of the AB oscillations in $G_1$, $G_2^S$ and $G_2^{AS}$ (ring$_2$). Continuous lines are the fits corresponding to the regions (1) $k_BT \ll E_c$ (2) $k_BT \gg E_c$ but $L \ll L_\phi$ (3) $L > L_\phi$.(b)Examples of temperature dependence of $G_2$ in ring$_2$ for different values of magnetic field between -1000 and 1000 Gauss.

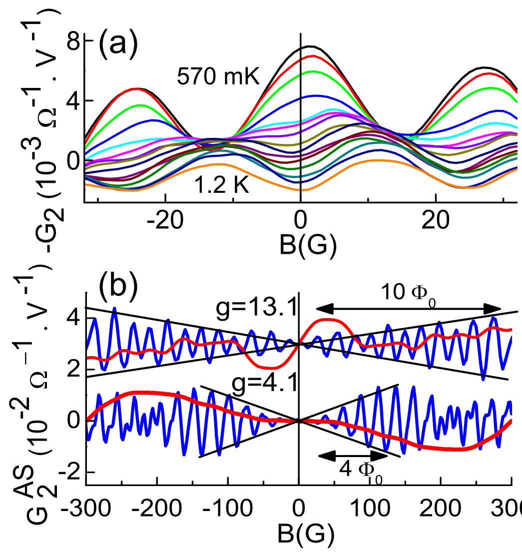


FIG. 4: (a)Low field dependence of $G_2$ on ring$_1$ for different temperatures.(b) Field dependence of $G_2^{AS}$ after low pass (light line) and high-pass (bold line) filtering , on ring$_1$ and ring$_2$ respectively lower and upper curves (shifted for clarity). Note the extinction of AB oscillations in the vicinity of $B = 0$.

larger dependence measured in $\delta G_2^S(g)$ indicates, according to (1), that $E_c = g\Delta$ decreases with $g$. This finding can be attributed to the physics involved in the illumination process which increases the width of the rings (resulting in a decrease of $\Delta$) and also decreases the elastic scattering time. Finally we find that the ratio $\delta G_2^{AS}/\delta G_2^S$ depends only slightly on $g$ and is equal to $0.3 \pm 0.2$(see Fig.2c). This result is *a priori* in contradiction with Eq.1 from which a $1/g$ dependence of $\delta G_2^{AS}/\delta G_2^S$ is expected. However Eq.1 is only valid for $\gamma_{int} \ll 1$ in partially closed quantum dots, i.e. whose classical resistance is dominated by the contacts but still much lower than quantum resistance. It was recently found by one of us [20] using random matrix theory that in these systems $\delta G_2^S$ also strongly depends on $\gamma_{int}$, when it is not negligible compared to 1, and in the limit $\Phi \gg \Phi_c$:

$$\delta G_2^{AS}/\delta G_2^S = 1/\sqrt{1 + 2(g/\delta G_1)^2(1/\gamma_{int} - 1)^2} \qquad (2)$$

identical to (1) in the limit of small $\gamma_{int}$. It is interesting to note that Eq.2 predicts $\delta G_2^{AS}/\delta G_2^S$ independent of $g$ in the limit $\gamma_{int} = 1$. It describes remarkably well our experimental results, yielding $\gamma_{int} \simeq 0.94 \pm 0.02$ for ring$_2$ and $\gamma_{int} \simeq 0.85 \pm 0.02$ for ring$_1$ . These values are very close to the estimated values of $\gamma_{int} = 1/(1 + C\Delta/2e^2) \simeq 0.98$ from the geometry of our samples. Our determination of the interaction constant from the dimension less quantity $\delta G_2^{AS}/\delta G_2^S$ is much more accurate than the estimation done in reference [17] from the analysis of the low field component of $G_2^{AS}$ only, but consistent with it. Finally, our results show that our rings are in a highly interacting regime and behave as partially closed quantum dots due to their relatively long mean free path.

The temperature dependence of $\delta\tilde{G}_1$ and $\delta\tilde{G}_2^{S,AS}$ are shown on Fig.3a. As already observed [22] $\delta\tilde{G}_1$ is only weakly $T$ dependent below the Thouless energy like $1 - AT^2$ with $A = 2(k_B/E_c)^2$ and decays at higher temperature as $T^{-1/2}$. Deviations above $0.5K$ are consistent with $\exp(-L/L_\phi(T))$ with a $T$ dependence of the phase coherence length as $T^{-1/2}$ but this last fit on a small range of temperature is not unique and just indicative. $\delta\tilde{G}_2^{S,AS}$ have similar $T$ dependences, which are nearly identical to $(\delta\tilde{G}_1(T))^2$ with a $T^{-1}$ decay in the limit $k_BT \gg E_c$ with $L \ll L_\phi$, in agreement with theoretical predictions [8, 11]. The temperature dependence of $G_2$ at fixed magnetic fields is depicted in Fig.3b. In the same way as observed on $G_1$ [23], $G_2(T)$ exhibits a non monotonous variation with temperature on the scale $E_c$ which randomly fluctuates with magnetic field. In some cases we also observed (see Fig.4a) that the phase of the AB oscillations in $G_2$ depends on temperature. Surprisingly it remains pinned either to 0 or $\pi$ at zero field with the appearance of a second harmonics contribution in the region of temperature where the sign change occurs. At larger field the phase takes any value between 0 and $\pi$. Note that this last effect is observed both in $G_1$ and $G_2$ as a result of phase modulation of the AB oscillations by the UCF, but these phase modulations are symmetric in field on $G_1$, and not on $G_2$. In short, we find that AB oscillations on $G_2$ are symmetric in zero field and the asymmetry only appears at higher field. This is also clearly seen in the AB oscillations on $G_2^{AS}$. We observe in all samples that their amplitude vanish linearly at zero field (Fig.4b).

We now propose a simple explanation of the extinction of AB oscillations at low field in $G_2^{AS}$ linked to the larger AB component observed in $G_2$ compared to $G_1$. In the semiclassical approximation ($k_f l_e \gg 1$ ) it is

possible to express the conductance of a phase coherent ring in terms of interference between scattering amplitudes of electronic waves, $G = \mathrm{Re}\,[\Sigma_{i,j} A_i A_j \exp i\phi_{ij}(B)]$, where indices $i$ and $j$ run on all pairs of possible diffusive trajectories going from one terminal to the other and $\phi_{ij}(B) = \phi_i - \phi_j + 2\pi BS_{ij}/\Phi_0$. The phase at zero field $\phi_i$ is the integral $(1/\hbar)\int_i U_{dis}(r(\vec{t}))dt$ on the diffusive trajectory $i$ assuming that the screened disordered potential $U_{dis}$ varies smoothly on the electron Fermi wavelength which is a reasonable assumption in a 2DEG. $S_{ij}$ is the surface comprised between trajectories $i$ and $j$. Onsager symmetry rules imply that the 2 terminal magnetoconductance takes the following form at zero temperature:

$$G{=}\Sigma_{i,j} A_i A_j \cos(\phi_{ij}) cos(2\pi BS_{ij}/\Phi_0) \tag{3}$$

In the presence of a current through the sample associated to a potential drop $V$, the local electronic density and consequently the scattering potential is modified with a term $dU_{dis}(\vec{r}, V)$ which induces phase shifts, $d\phi_{ij} = \int_i - \int_j (1/\hbar) dU_{dis}(r(\vec{t}))dt$. The quantity $G_2 = (dG(V)/dV)_{V=0}$ is then directly related to these phase shifts through:

$$G_2 = \Sigma_{i,j} A_i A_j (d\phi_{ij}/dV) \sin(\phi_{ij}) \cos(2\pi BS_{ij}/\Phi_0) \tag{4}$$

Since $d\phi_{ij}$ increases with the length of the interfering trajectories $i$ and $j$, long trajectories encircling the ring (AB oscillations) contribute more than trajectories within the same branch of the ring (UCF). This explains the larger relative amplitude of the AB oscillations and the larger harmonics content in $G_2$ compared to $G_1$ (see Fig.2 a,b). In a diffusive open system $dU_{dis}$ is of the order of $eV$ and the main contribution to the typical value of $d\phi_{ij}$ is $eV\tau_D/\hbar$ and is independent of B leads to the expression of $\delta G_2^S$ of Eq.1. This contribution is strongly attenuated for a ballistic system with strong interactions, like in [11], where most of the potential drop takes place at the contacts. Other contributions to $\delta G_2$ of the order of 1/g are obtained by taking into account mesoscopic non local field dependent fluctuations in the potential $dU_{dis}(\vec{r}, B)$, with symmetric and antisymmetric components in B, both of order of $\gamma_{int} eV \delta G_1/g$ at high flux compared to $\Phi_c$. The antisymmetric component in $dU_{dis}(\vec{r}, B)$ is expected to typically vary like $\gamma_{int}(\delta G_1/g)V\Phi/\Phi_c$ at low flux. We can then deduce from Eq.4 the main contributions to the AB oscillating part of $\tilde{G}_2^{AS}$, which originates from pairs of trajectories encircling the rings:

$$\tilde{G}_2^{AS} = \frac{\delta G_1}{g\hbar}\gamma_{int}\Sigma_{i,j} e A_i A_j \tau_{ij} \sin(\phi_{ij}) \frac{\Phi}{\Phi_c} \cos(2\pi\Phi/\Phi_0) \tag{5}$$

There is also a smaller contribution of terms in $\sin(2\pi\Phi/\Phi_0)$ of the order of $\delta\tilde{G}_1/\delta G_1$ which is less than 0.2 in our experiments. This provides an explanation for the linear increase of the amplitude of the AB oscillations of $G_2^{AS}$ in a flux range $\Phi_c$ for rings with $g > 1$ as shown in Fig.4b. However this effect may not exist in very narrow rings [18] where conductance fluctuations are not observed. It is interesting that this simple heuristic model can also explain the larger value of $G_2^S$ in a diffusive compared to a ballistic system and the different values of the ratio $G_2^{AS}/G_2^S$ of the order of $\gamma_{int}/g$, respectively 1, in a diffusive, respectively ballistic, system.

In conclusion we have shown evidence of a field asymmetry on the second order response of GaAs/GaAlAs rings of mesoscopic origin which contains both AB oscillations and conductance fluctuations. This asymmetry is characterized by $\delta G_2^{AS}/\delta G_2^S$ and analyzed within theoretical predictions expressing this ratio with only 2 parameters, the dimensionless conductance of the rings and the interaction constant whose value can be determined $\gamma_{int} = 0.90 \pm 0.05$. We have also found that the relative amplitude of the AB oscillations compared to the UCF is much larger in $G_2$ than in $G_1$ with the existence of a linear low field modulation in the AB oscillations in the antisymmetric component of $G_2$. These effects can be understood within a simple semi-classical description of quantum interference. We thank D. Mailly and F. Pierre for their helpful contribution to the fabrication of the samples as well as M. Büttiker, M.Ferrier, G. Montambaux, B.Reulet, B. Spivak and C. Texier for fruitful discussions. *Note added* During completion of this work, related experimental work on gated quantum dots [17] and small Aharonov Bohm rings [18] appeared.


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